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The upper limit to the creep life of solids under a tensile force

B. BURTON
Central Electricity Generating Board, Berkeley Nuclear Laboratories, Berkeley, Gloucestershire, UK

Creep failure generally occurs either by the attenuative loss of area as flow proceeds (geometrical failure) or by the development of internal cavities. The former mechanism dictates the ultimate creep life of a material. Criteria are developed for this upper limit to life for a variety of creep mechanisms. The effects of both primary creep and necking are discussed and a brief comparison is made between failure criteria for cavitation failure and geometrical failure.

1. Introduction

When a solid fails to creep, the failure mechanism falls into one of two main categories. The first, called in this paper geometrical creep rupture, GCR, depends only on the geometry of flow and on the form of the flow equation. In tension, this type of failure occurs simply because cross-sectional area is lost during flow in order to preserve constant volume. Flow may be uniform almost up to the point of failure but is often associated with local necking towards the end of life. Failure strains by this mechanism are relatively high, simply because significant changes in geometry are required.

The second main type of failure process occurs in polycrystalline materials by the nucleation, growth and coalescene of cavities on grain boundaries. These cavities may be rounded in appearance, indicating that diffusion processes are important; or they may be wedge shaped and typical of a growth which involves gross separation or relative movement of adjacent grains. The relative importance of the two cavity types depends on microstructures, stress and temperature [1].

A substantial amount of effort has been directed towards the understanding of the precise microscopic details of cavity growth, including considerations of geometry, stress re-distribution and surface diffusion aspects; although relatively little attention has been given to the (albeit much simpler) criterion for geometrical creep rupture. Such a criterion is however very important since it represents the ultimate creep life of any solid. GCR occurs when the area has decreased by flow to such an extent that it can no longer carry the applied load. If any other processes are operating (for example cavitation or necking) they can only shorten the creep life. No mechanism can extend it. An extension in creep life can only be achieved through a change in the flow equation.

Thus the GCR criterion defines an upper limit to creep life and represents a useful first step in analying creep failure data. If experimental data falls too close to these predictions (and they often do), then further involved speculations about the failure mechanism may not be necessary. Even if cavities are present, they need not contribute significantly to the overall failure process in many instances.

2. Geometrical creep rupture

When any solid undergoes steady state irreversible flow under the action of an applied stress, τ, the flow rate, $\dot{\gamma}$, is given by the relationship

$$\dot{\gamma} = \sum \alpha_i (\tau - \beta_i)^{n_i}. \quad (1)$$

This equation represents the sum of all mechanisms, i, operating, where α_i, β_i and n_i depend on the mechanism. For crystalline solids $1 \leqslant n \leqslant 5$ is a numerical constant, $\beta \geqslant 0$ is a threshold stress below which steady state flow does not occur and α contains physical, thermodynamic and structural parameters appropriate to the material. Some examples for crystalline materials are shown in Table I.

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Geometrical creep rupture depends on the parametric form of the flow equation, Equation 1. It is appropriate to solids creeping under the action of a constant tensile force and arises because of the unstable nature of such flow. For this type of failure a rigorous theoretical upper bound to the creep life of any solid can be defined.

For a unaxial tensile force, F, the creep equation can be written as

$$\frac{d\epsilon}{dt} = \frac{1}{l}\frac{dl}{dt} = -\frac{1}{A}\frac{dA}{dt} = a\left(\frac{F}{A} - \sigma_0\right)^n, \quad (2)$$

where $(d\epsilon/dt)$ is the instantaneous creep rate, l and A are the specimen length and area and σ_0 is a threshold stress appropriate if the material is a dispersion hardened crystal. It is often close to the Orowan stress, Gb/λ, where G is the shear modulus, $\boldsymbol{b}$ is the Burgers vector and λ is the inter-particle spacing. The rate of loss of specimen area is thus

$$-\left(\frac{dA}{dt}\right) = \frac{aF^n}{A^{n-1}}\left(1 - \frac{\sigma_0}{F}A\right)^n. \quad (3)$$

This equation can be integrated, for a constant applied force, F, using the boundary conditions $A = A_0$ at $t = 0$ to give the variation of area with time. Thus

$$(A_t/A_0)^n = 1 - \frac{naF^n t}{A_0^n} + n^2 \sum_{j=1}^{\infty} \frac{(n+j-1)!}{n!j!(n+j)} \times (\sigma_0/\sigma)^j \left[1 - (A_t/A_0)^{n+j}\right]. \quad (4)$$

The failure condition is when the ratio of the creep ratio at time, t, to the steady-state creep rate approaches infinity. That is when $(A_t/A_0) \to 0$. The failure condition is thus

$$n\dot{\epsilon}t_f = \left[1 - \frac{\sigma_0}{\sigma}\right]^n \left[1 + n^2 \sum_{j=1}^{\infty} \frac{(n+j-1)!}{n!j!(n+j)} \left(\frac{\sigma_0}{\sigma}\right)^j\right], \quad (5)$$

where $\dot{\epsilon}$ and σ are respectively the initial creep rate and the initial stress.

3. The limiting cases

The flow of solids follows general equations which are termed viscous flow, power law creep and threshold flow. These are now considered generally and specific examples are then given for polycrystalline metals.

3.1. Viscous flow, $\sigma_0 = 0, n = 1$

This type of flow has the form

$$\dot{\epsilon} = a_1 \sigma, \quad (6)$$

where a_1 can be obtained from Table I. Substituting $\sigma_0 = 0$ and $n = 1$ into Equation 4 gives the variation of area with time: Equation 2 then gives the variation of creep rate with time. The strain–time curve is then obtained by integration and is given by

$$\epsilon_t = -\ln(1 - \dot{\epsilon}t), \quad (7)$$

or, in terms of creep rate

$$\frac{\dot{\epsilon}_t}{\dot{\epsilon}} = \frac{1}{1 - \dot{\epsilon}t}. \quad (8)$$

TABLE I Approximate creep parameters in the equation $\dot{\epsilon}kT/D_E Gb = A_c(\sigma/G) - (\sigma_0/G)^n$

Creep		n	A_c	σ_0	D_E	Reference
Diffusion creep:	Single phase	1	$10(b/d)^2$	Small	$D_L\, 1 + \frac{15}{\pi}\frac{w}{d}\frac{D_G}{D_L}$	[2]
	Dispersion hardened	1	$10(b/d)^2$	Proportional to volume-fraction	$D_L\, 1 + \frac{15}{\pi}\frac{w}{d}\frac{D_G}{D_L}$	[2]
Recovery creep:	Single phase	3	1	0	$D_L\, 1 + \left(\frac{\sigma}{G}\right)^2 \frac{D_P}{D_L}$	[1]
	Dispersion hardened	3	1	b/λ	$D_L\, 1 + \left(\frac{\sigma}{G}\right)^2 \frac{D_P}{D_L}$	[1]
	Solute drag	3	1	0	D_S	[1]
Superplastic	Duplex structure	1–3	100	Interpretation varies (see text)	Composite alloy diffusion coefficient	[3]

λ is the inter-particle spacing; d is the grain size; D_L is the lattice diffusivity; D_G is the grain boundary diffusivity; D_S is the solute diffusivity; and D_P is the dislocation pipe diffusivity.

The failure time is when this ratio becomes infinite, that is:

$$\dot{\epsilon} t_f = 1, \tag{9}$$

where t_f is the time to failure.

In principle, viscous solids should flow almost indefinitely since the rate of reduction of area, $-(dA/dt)$, is independent of area. Thus, even incipient necks should not develop since the material can maintain a constant ratio of neck size to component size independent of specimen strain. Such behaviour is typical of glasses at high temperature, amorphous materials like pitch, and certain plastics.

Polycrystalline materials can also deform in a way in which creep rate and stress are linearly related. This mechanism is diffusional creep (see Table I). In principle these materials should also exhibit exceptional ductilities. However in practice, the rates of strain are often so low that the predicted failure times are very long. During this time grain coarsening often occurs, thus reducing rates still further, and in certain materials (particularly ceramics), cavity development may contribute to failure. (It should also be noted that if the cross-sectional area does decrease significantly during diffusional flow, the corresponding stress increase may cause dislocation processes to become operative in the later stages.) For metals in service, rupture by this mechanism is not to be expected by diffusional creep.

3.2. Power-law creep, $\sigma_0 = 0, n > 1$

The power-law creep flow equation has the form

$$\dot{\epsilon} = a_2 \sigma^n. \tag{10}$$

Following a similar procedure, the relationships between strain and time are found to be

$$\epsilon_t = -\frac{1}{n} \ln(1 - n\dot{\epsilon}t) \tag{11}$$

and

$$\dot{\epsilon}_t = \dot{\epsilon}/(1 - n\dot{\epsilon}t). \tag{12}$$

The failure criterion is once again when $(\dot{\epsilon}_t/\dot{\epsilon}) \to \infty$ that is:

$$n\dot{\epsilon}t_f = 1. \tag{13}$$

Thus, the time to failure by the GCR mechanism for any particular creep rate depends only on the stress index, n.

Pure metals at intermediate stress levels creep by a power law mechanism. When dislocation glide is much easier than climb, creep is diffusion controlled (recovery creep) and obeys the relationship (given in Table I):

$$\frac{\dot{\epsilon}kT}{DGb} = A_c\left(\frac{\sigma}{G}\right)^3\left\{1 + \left(\frac{\sigma}{G}\right)^2 \frac{D_p}{D}\right\}, \tag{14}$$

where D_p is the dislocation pipe diffusion coefficient. The creep constant $A_c \sim 1$. Thus the stress time to rupture relationship is

$$t_f = \frac{kT}{nDGb(\sigma/G)^3} \tag{15}$$

when lattice diffusion gives the greatest contribution, and

$$t_f = \frac{kT}{nD_pGb(\sigma/G)^5} \tag{16}$$

when dislocation pipe diffusion predominates. Since the activation energy for pipe diffusion is less than that for lattice diffusion and the stress dependence is different in Equations 15 and 16, a transition in the fracture criterion is expected with Equation 16 being appropriate at lower temperatures and higher stress levels.

In alloys which are strengthened by solutes, or in pure metals which have an inherently low stacking fault energy, creep rates are lower than those predicted by Equation 14. That is, the creep constant A_c is lower in both the above types of material, and in addition for solute drag controlled creep the transition to pipe diffusion is suppressed. This is because dislocation movement is impeded by the drift of the solute "atmosphere" around dislocations and such solute movement has to occur by lattice diffusion. Thus t_f by the GCR mechanism is longer for these materials and this once again emphasizes that the time to failure depends only on the form of the flow equation.

3.3. Threshold flow, $\sigma_0 > 0, n \geq 1$

Certain materials exhibit a threshold stress below which flow does not occur. When $n = 1$ this type behaviour is known as Bingham flow. In polycrystalline materials it is typical of diffusional creep when a dispersion of refractory second phase particles are present on grain boundaries and is also characteristic of a range of non-metallic solids. From Equation 5, the failure criterion is:

$$\dot{\epsilon}t_f = \left(1 - \frac{\sigma_0}{\sigma}\right)\left\{1 + \sum_{j=1}^{\infty} \frac{1}{(1+j)}\left(\frac{\sigma_0}{\sigma}\right)^j\right\}. \tag{17}$$

When $n \geq 1$ the flow equation is typical of dispersion hardened crystalline materials, where the threshold stress can be correlated with the Orowan stress Gb/λ, where λ is the dispersion spacing. In this case the flow behaviour is more complex. Equation 5 represents the failure criterion which is shown graphically in Fig. 1. The parameter $n\dot{\epsilon}t_p$ is plotted against stress, normalized in terms of the threshold stress σ/σ_0 for various values of n. The curves become asymptotic to the criterion $n\dot{\epsilon}t_f = 1$ at large values of applied stress and equal to $n\dot{\epsilon}t_f = 0$ at $\sigma = \sigma_0$.

Using the expression for the creep of dispersion hardened crystalline materials given in Table I, then

$$t_f = \frac{kT}{nDGb}\left\{\frac{1+n^2f}{(\sigma/G)^n}\right\}, \tag{18}$$

where f is the summation of Equation 5 whose values can simply be obtained from Fig. 1. The failure times for a dispersion hardened material are shown in Fig. 2. The calculation is for values of σ_0 appropriate to an Orowan stress Gb/λ for interparticle spacings of ∞, 10, 3 and 1 μm respectively. It is clear that the rupture life is extended dramatically by the presence of second phase particles which are effective in providing a threshold stress.

3.4. Superplastic flow

It has been noted that whilst materials deforming by diffusional creep should in principle extend indefinitely, in practice they often do not owing to other contributing factors. The GCR criterion does, however, have particular significance in superplastic flow. This type of flow, like diffusional creep, occurs at relatively low stress levels. Whilst the mechanism is not fully understood, it is clear that diffusional creep plays an important role. The stress dependence of superplastic creep does not usually approach the limiting value of unity, but its value is sufficiently low that the product $\dot{\epsilon}t_f$ is high enough to give rise to extensive ductility by the GCR mechanism.

The importance of the GCR mechanism in determining ductility can clearly be seen by a closer inspection of the stress dependence of superplastic creep. This dependence is sigmoidal and is represented schematically in Fig. 3. At high rates of strain, behaviour is characteristic of recovery creep processes where creep rate and stress are related by a power law. Thus, in this region $\dot{\epsilon}t_f$ has a relatively low value. This value has the limit $1/n$ for the GCR mechanism but may be even lower if cavitation failure mechanisms are operating. At lower stresses behaviour

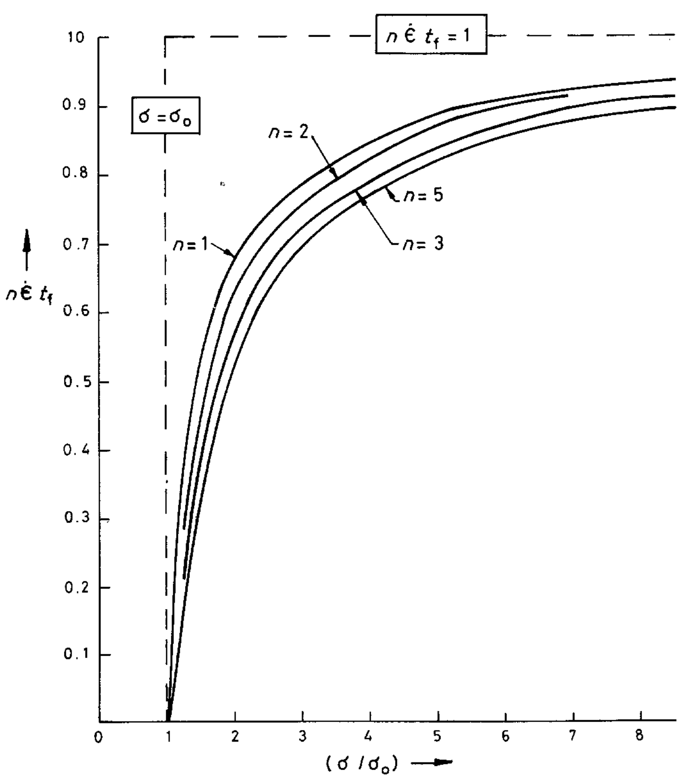

Figure 1 Variation of the creep rupture parameter with stress for any flow equation of the form $\dot{\epsilon}\alpha(\sigma-\sigma_0)^n$. t_f is the failure time.

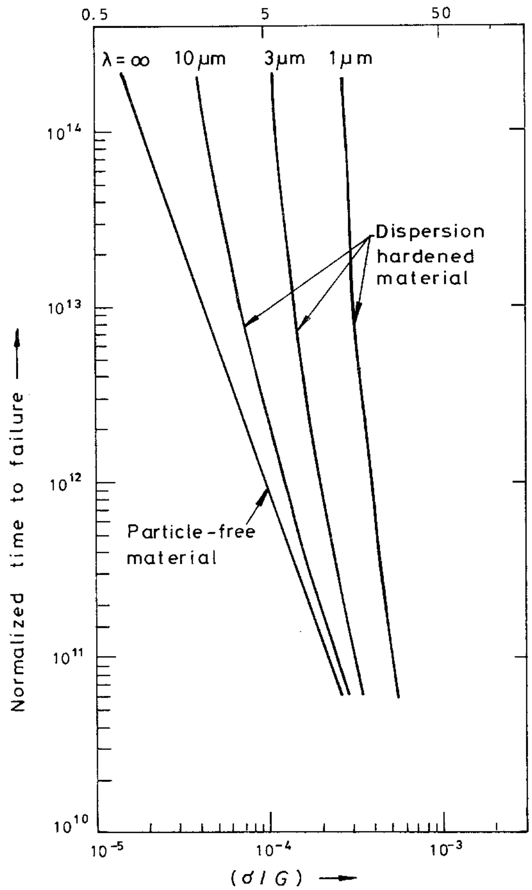

Figure 2 Variation of the normalized time to failure $DGbt_f/kT$ with stress for a dispersion-hardened crystal containing a range of particle spacings.

is more complex. The stress dependence decreases and then increases once more at the lowest stress levels. This behaviour has been interpreted in terms of the operation of two independent mechanisms, or alternatively as being typical of a material which exhibits a threshold stress. These alternative explanations have important consequenes as far as the understanding of microspoic creep mechanisms is concerned; but for the present purpose which is concerned primarily with the geometry of flow, they are relatively unimportant. It is the overall flow characteristics of the material which are important in determining the GCR criterion, not the microscopic details of the creep mechanism. Also shown in Fig. 3 is the variation of the parameter $\dot{\epsilon}t_f$. The region of maximum ductility corresponds to the maximum value of this parameter.

4. Creep failure by cavitation

The main aim of this paper is to indicate the upper limit to creep life of a solid. GCR represents such an upper limit. If any other failure mechanism operates, creep life can only be shorter than this upper limit. No mechanism can extend it. Cavitation development on grain boundaries during creep is one mechanism which can shorten creep life.

Diffusion cavity growth depends on a diffusive flux of vacancies from a stressed boundary to the surface of a suitable cavity nucleus on the boundary. If vacancies are being supplied by grain boundary diffusion (coefficient D_G) the radial growth rate of such a cavity has been calculated by Hull and Rimmer [4] to be:

$$\frac{dr}{dt} = \frac{4\pi\sigma\Omega D_G\omega}{\lambda rkT}, \qquad (19)$$

where Ω is the atomic volume, λ the inter-cavity spacing, r is the radius and $\omega \sim 2b$ is the boundary width. Since the development of this equation many refinements have been introduced (see the review of Beeré [5] for example) which are beyond the scope of discussion in this paper. In essence, however, a failure time by cavitation can be arrived at by integrating the above equation to obtain an expression for the variation of cavity radius with time and writing the failure time as the time for the cavities to grow to a critical size. At a simple level this critical size may be assumed to be $r_{crit} = \lambda/2$. That is, the material fails when the cavities touch. The material between the cavities starts to creep locally long before the above condition is reached, however, a more sophisticated criterion which includes a stress-dependent critical size is required. At this critical size, cavities begin to coalesce by the local necking or plastic opening of material between the cavities, and the cavitated boundary takes on the characteristics of a wedge-shaped crack. Once this situation obtains, opening continues by local plastic flow or gross movement of adjacent grains by a boundary sliding process.

Once wedge-type cracks have formed, the residual creep life becomes exhausted in a way which resembles GCR. In GCR, cross-sectional area is being lost by bulk flow so that opposite faces of the specimen approach the specimen axis to preserve constant volume, whereas for wedge crack opening area is being lost internally by plastic opening and/or sliding. The point is that in both cases area is being lost by a process of plastic creep (or boundary sliding which is in

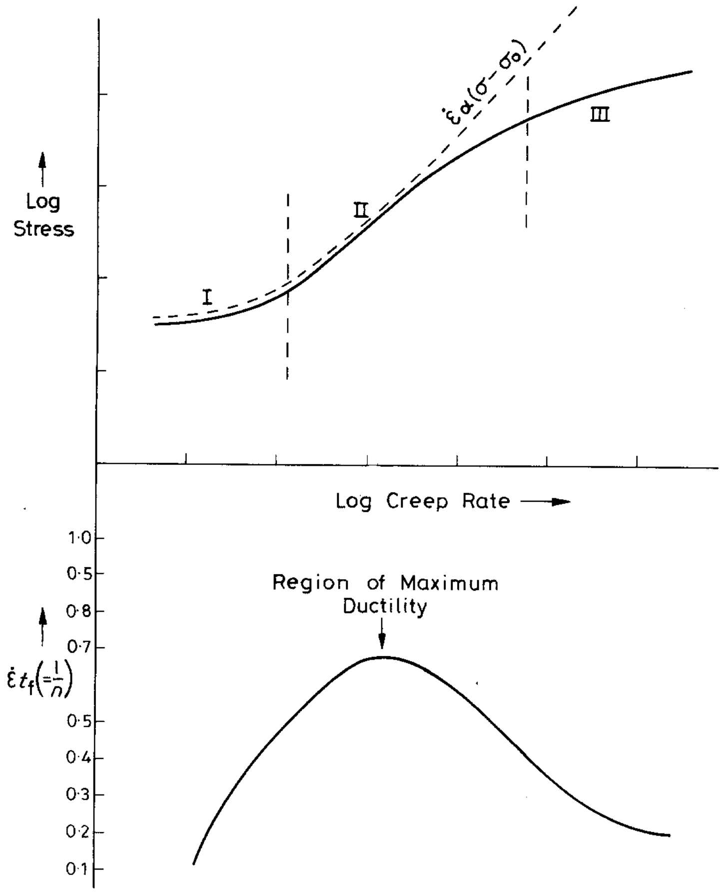

Figure 3 The sigmoidal variation of creep rate and stress shown schematically for a superplastic material. The creep mechanism at low stresses may be interpreted in terms of two mechanisms called I and II or in terms of a threshold stress, σ_0. The main point as far as GCR is concerned is that the apparent stress exponent $(d \log \sigma)/(d \log \dot{\epsilon})$ reaches a maximum at some intermediate value of stress.

itself related to creep), so that the failure criteria are fairly similar. An approximate calculation of the time to failure by boundary sliding is presented in the Appendix.

5. The influence of primary creep

The calculations are concerned entirely with the equations for steady-state creep. Real materials usually show a transient stage which may persist to strains of 0.1 or more. As already mentioned, primary creep is of little importance in the dispersion hardened material but is significant for the softer material. The development of overall creep strain with time can be described by the expression:

$$\epsilon = \epsilon_0 + \epsilon_T \, [1 - \exp(-mt)] + \dot{\epsilon}_{ss} t, \quad (20)$$

where ϵ_0 is the initial or instantaneous extension on loading, ϵ_T is the total primary creep strain, m is the primary creep rate constant, $\dot{\epsilon}_{ss}$ is the steady-state creep rate at constant stress and t is the time. The rate of creep is obtained by differentiating. Thus:

$$\dot{\epsilon} = m \, \epsilon_T \exp(-mt) + \dot{\epsilon}_{ss}. \quad (21)$$

Since the time to failure is a function of the flow rate, an effective rate of creep, $\bar{\dot{\epsilon}}$, can be calculated, given by

$$t_f(\bar{\dot{\epsilon}}) = m \, \epsilon_T \int_0^{t_f} \exp(-mt) \, dt + \dot{\epsilon}_{ss} \int_0^{t_f} dt. \quad (22)$$

Using the failure criterion:

$$n \, (\bar{\dot{\epsilon}}) \, t_f = 1 \quad (23)$$

and the empirical relationship [3]:

$$m = \alpha \, \dot{\epsilon}_{ss}, \qquad \alpha = 10 \rightarrow 100 \quad (24)$$

gives the relationship between t_f and $\dot{\epsilon}_{ss}$ to be

$$\alpha \, \dot{\epsilon}_{ss} \, t_f = -\ln \left(1 + \frac{\dot{\epsilon}_{ss} \, t_f \, n - 1}{\epsilon_T n} \right). \quad (25)$$

The numerical solution of Equation 25 is shown in graphical form in Fig. 4, for different values of ϵ_T (= 0.1 and 0.2) and α (= 10 to 100). It is clear that materials showing extensive (large

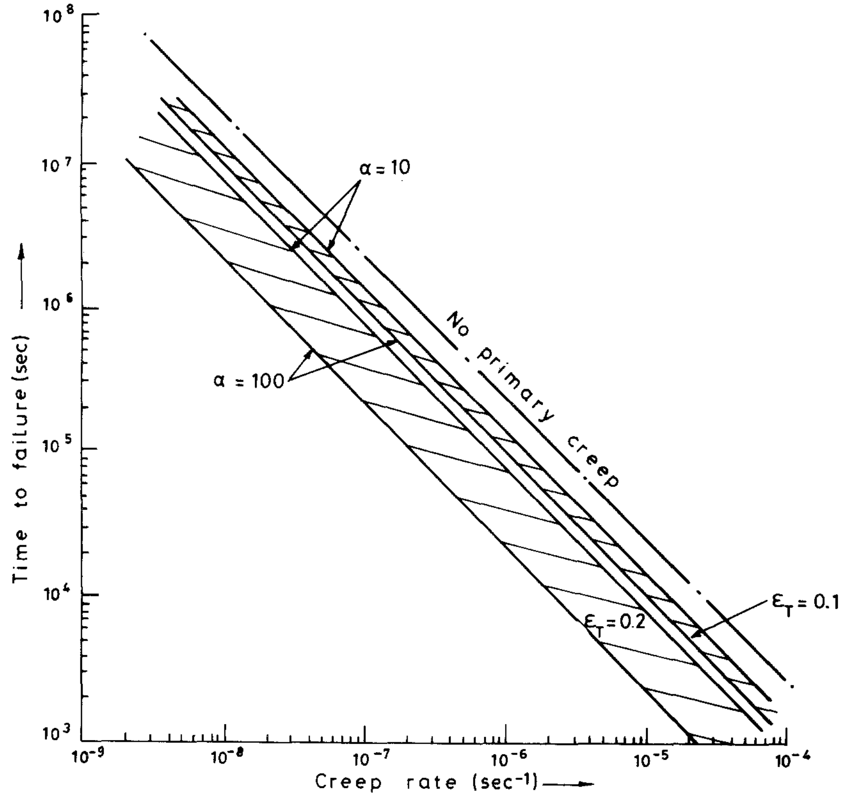

Figure 4 The influence of primary creep on creep rupture. Typical parameters in the primary creep equations are shown.

ϵ_T) or rapid (large m) primary creep will have a significantly reduced time to failure compared to that calculated from the steady-state rate. This is simply because they have spent a fraction of their lives deforming at fast rates and the failure time is only a function of rate and not of mechanism so long as the stress dependence remains unchanged.

6. Discussion

The type of rupture process considered in this paper is the simplest type which can occur during creep. It depends only on the attenuation of cross-sectional area which enables a constant specimen volume to be maintained during flow. The failure criterion is dictated solely by the overall characteristics, i.e., the parametric form of the flow equation and not by the microscopic details of some internal mechanism. The creep life which is calculated represents a rigorous upper limit to the creep life of solid, and the product $n\dot{\epsilon}t_f$ may also be regarded as a ductility parameter. No material can exhibit a value of the parameter greater than unity.

If solids fail with a value of $n\dot{\epsilon}t_f$ less than predicted in this paper, this may indicate that other fracture processes are operative. These other processes can be contributions by necking or primary creep. The necking contribution is not a separate failure mechanism; it simply reflects the fact that for power-law creep, the rate of loss of area $(-\mathrm{d}A/\mathrm{d}t)$ is in itself a function of the instantaneous area (Equation 3) so that any surface perturbation or incipient neck becomes increasingly exaggerated during subsequent flow. Similarly, primary creep gives rise to values of $n\dot{\epsilon}t_f$ less than predicted by writing $\dot{\epsilon}$ as the steady-state creep rate. The shorter life simply arises since the specimen has spent a part of this life at a creep rate greater than the steady-state rate.

Whilst necking and primary creep contributions are simply extensions to the attenuation mechanism of rupture, internal void and crack development must be considered as mechanisms independent of this. These cavitation mechanisms depend, at least initially, on diffusive growth. They are thus likely in materials where the relative rates of diffusion of those of creep are high. Examples are the creep of solute drag controlled alloys, where dislocation movement if limited by the lattice diffusion solute atmospheres. In such alloy, relatively rapid growth of cavities can occur by short circuiting diffusion and thus cavitation will become increasingly important at lower temperatures. Cavitation is also more likley in metals and alloys with low stacking fault energy. These alloys creep at reduced rates whilst the diffusive processes responsible for cavity growth are unaffected by SFE. Thus aluminium (high SFE) fails by GCR and never cavities. Silver, on the other hand, which has the same fcc structure but low SFE is often used as a model material to demonstrate cavity growth.

7. Conclusions

The simplest form of tensile creep failure occurs by the attenuation of cross-sectional area which is necessary to conserve material volume during flow.

A failure criterion can be developed which relates the flow rate of the material to a failure time which represents the ultimate creep life attainable.

The failure time may be less than this value if necking occurs or if the material shows a primary creep stage.

Failure by cavitation development is an independent mechanism to the geometrical rupture mechanism and may give rise to substantially lower creep life. However, if wedge-type cavities develop by grain boundary sliding, the failure criterion may be similar in form to that for geometrical failure since in both cases area is being lost in a way which depends on creep rate.

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Appendix: The failure time by wedge-crack opening

A wedge-shaped crack of length c on a grain-boundary facet of length l and unit thickness is considered to be opening by a grain-boundary sliding process as indicated in Fig. A1. The opening rate of the crack, $\mathrm{d}c/\mathrm{d}t$, is assumed to be equal to the sliding rate of the boundary and the sliding rate itself to be related to the creep rate. Thus,

$$\frac{\mathrm{d}c}{\mathrm{d}t} = \alpha l \dot{\epsilon}, \qquad \text{(A1)}$$

where α is a constant containing angular terms. It is assumed to be unity in this calculation. The local creep rate is given by

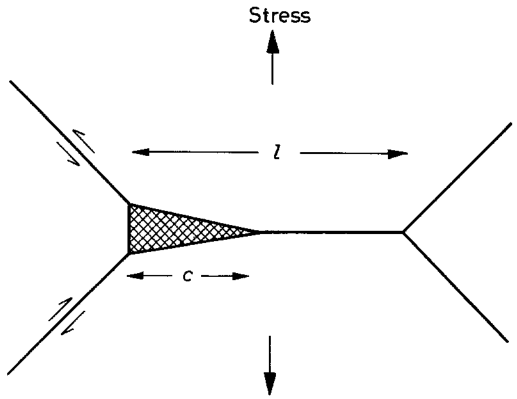

Figure A1 Schematic diagram of a wedge crack opening by grain-boundary sliding.

$$\dot{\epsilon} = A_c \left[\frac{\sigma}{1-(c/l)}\right]^n, \qquad \text{(A2)}$$

where the term $1-(c/l)$ accounts for the loss of cross-sectional area as the crack opens. The failure criterion is clearly

$$\dot{\epsilon} t_f = \frac{1}{l}\int_{c_0}^{c_f}\left(1-\frac{c}{l}\right)\mathrm{d}c = \frac{1}{n+1}, \qquad \text{(A3)}$$

where c_0 is the initial crack length (assumed to be zero) and $c_f \to l$ is the crack length at the failure time, t_f. This wedge-cracking criterion is only less than the GCR criterion by the factor $1 + (1/n)$, which is about unity for large n. The important point to note is the parametrically similar form of the two criteria. They arise in both cases since creep rate is increasing because of loss of area (either total area by attenuation or by local internal cracking) and this loss of area in itself if controlled by the rate of creep.

References

1. J. H. GITTUS, "Creep, Viscoelasticity and Creep Fracture in Solids" (Applied Science Publishers, New York, 1975).
2. B. BURTON, "Diffusional Creep of Polycrystalline Materials" (Trans. Tech. Switzerland, 1977) p. 12.
3. J. E. BIRD, A. K. MUKHERJEE and J. E. DORN, Proceedings of the International Conference on the Quantitative Relation between Properties and Microstructure (Israel University Press, Haifa, 1969) p. 255.
4. D. HULL and D. E. RIMMER, *Phil. Mag.* 4 (1959) 673.
5. W. B. BEERÉ, in "Cavities and Cracks in Creep and Fatigue", edited by J. H. Gittus (Applied Science Publishers, New York, 1981).

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